Factoring Overview

Is there a Common Factor?
  Yes → Factor out the GCF
  No

Are there 2 terms?
  Yes → Is it a difference of squares?
  Yes → \((a+b)(a-b)\)
  No
  No → Is it a difference of cubes?
  Yes → \((a-b)(a^2+ab+b^2)\)
  No → Is it a sum of cubes?
  Yes → \((a+b)(a^2-ab+b^2)\)
  No → Prime

Are there 3 terms?
  Yes → Is it a perfect square trinomial?
  Yes → \((a+b)^2\) or \((a-b)^2\)
  No → Use trial and error or ac method
  No

Are there 4 or more terms?
  Yes → Use factoring by grouping
### Examples

Factor:

1. \[3x^2 - 12y^2\]

\[
= 3 \cdot x^2 - 3 \cdot 4y^2 \quad \rightarrow \quad \text{Factor out the GCF}
\]

\[
= 3(x^2 - 4y^2) \quad \rightarrow \quad \text{2 terms are shown}
\]

\[
= 3[(x)^2 - (2y)^2] \quad \rightarrow \quad \text{Difference of two squares: } a^2 - b^2
\]

\[
= 3(x - 2y)(x + 2y) \quad \rightarrow \quad (a - b)(a + b)
\]

2. \[4x^3 + 32y^3\]

\[
= 4 \cdot x^3 + 4 \cdot 8y^3 \quad \rightarrow \quad \text{Factor out the GCF}
\]

\[
= 4(x^3 + 8y^3) \quad \rightarrow \quad \text{2 terms are shown}
\]

\[
= 4[(x)^3 + (2y)^3] \quad \rightarrow \quad \text{Sum of cubes: } a^3 + b^3
\]

\[
= 4(x + 2y)(x^2 - 2xy + 4y^2) \quad \rightarrow \quad (a + b)(a^2 - ab + b^2)
\]

3. \[8t^2 + 24t + 18\]

\[
= 2 \cdot 4t^2 + 2 \cdot 12 \cdot t + 2 \cdot 9 \quad \rightarrow \quad \text{Factor out the GCF}
\]

\[
= 2(4t^2 + 12t + 9) \quad \rightarrow \quad \text{3 terms are shown}
\]

\[
= 2[(2t)^2 + 2(2t) \cdot 3 + 3^2] \quad \rightarrow \quad \text{Perfect square trinomial: } a^2 + 2ab + b^2
\]

\[
= 2(2t + 3)^2 \quad \rightarrow \quad (a + b)^2
\]
4. $2x^2 + x - 15$

$= 2x^2 - 5x + 6x - 15$

$= (2x^2 - 5x) + (6x - 15)$  \[\text{Factor by grouping}\]

$= (2x^2 - 5x) + (3 \cdot 2 \cdot x - 3 \cdot 5)$  \[\text{Factor out the GCF}\]

$= x(2x - 5) + 3(2x - 5)$  \[\text{Factor out the GCF}\]

$= (x + 3)(2x - 5)$

$a = 2, c = -15$

$a \cdot c = 2(-15) = -30$

<table>
<thead>
<tr>
<th>Factors of -30</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1 \cdot 30$</td>
<td>29</td>
</tr>
<tr>
<td>$-2 \cdot 15$</td>
<td>13</td>
</tr>
<tr>
<td>$-3 \cdot 10$</td>
<td>7</td>
</tr>
<tr>
<td>$-5 \cdot 6$</td>
<td>1</td>
</tr>
</tbody>
</table>

5. $6x^2 - 5x - 4$

$= 6x^2 + 3x - 8x - 4$

$= (6x^2 + 3x) - (8x + 4)$  \[\text{Factor by grouping}\]

(be careful of negative sign!)

$= (3 \cdot 2x^2 + 3x) - (4 \cdot 2x + 4)$  \[\text{Factor out the GCF}\]

$= 3x(2x + 1) - 4(2x + 1)$  \[\text{Factor out the GCF}\]

$= (3x - 4)(2x + 1)$

$a = 6, c = -4$

$a \cdot c = 6(-4) = -24$

<table>
<thead>
<tr>
<th>Factors of -24</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(-24)</td>
<td>-23</td>
</tr>
<tr>
<td>2(-12)</td>
<td>-10</td>
</tr>
<tr>
<td>3(-8)</td>
<td>-5</td>
</tr>
<tr>
<td>4(-6)</td>
<td>-2</td>
</tr>
</tbody>
</table>

6. $15x^2 + 18xy - 5xt - 6ty$

$= (15x^2 + 18xy) - (5xt + 6ty)$  \[\text{Factor by grouping}\]

(be careful of negative sign!)

$= (3 \cdot 5 \cdot x \cdot x + 3 \cdot 6 \cdot x \cdot y) - (5 \cdot x \cdot t + 6 \cdot t \cdot y)$  \[\text{Factor out the GCF}\]

$= 3x(5x + 6y) - t(5x + 6y)$  \[\text{Factor out the GCF}\]

$= (3x - t)(5x + 6y)$
Factoring Exercises

Greatest Common Factor
1. 18x - 24 2. 50x^2y^2 + 35x^3y 3. 36a^4b + 45a^5b^4 + 81a^3b^2 4. x(a + 5) - y(a + 5)

Difference of Squares
5. i^2 - 25 6. 12x^2 - 27y^2 7. 75A^2v - 147i^2v 8. 2x^4 - 512

Difference of Cubes
9. p^3 - 64 10. 3a^3 - 24b^3 11. 5x^3y - 40y^4 12. 125x^3 - 216y^6

Sum of Cubes
13. 8a^3 + 64 14. 128x^3 + 54 15. 1000t^3 + 27u 16. 64x^3 + 343y^6

Perfect Square Trinomials
17. a^2 + 14a + 49 18. 16u^2 + 8u + 1 19. t^2 - 12t + 36 20. 16x^2 - 24xy + 9y^2

ac Method
21. x^2 - 4x - 12 22. 3k^2 + 4k - 4 23. 8a^2 - 10ab - 3b^2 24. 6m^2 - 19mn + 10n^2

Factor by Grouping
25. 6 + 3m + 2p + mp 26. 20 + 5s + 12t + 3st 27. 4 - 2a - 6b + 3ab 28. 5 + x - 5y - xy

Factor:
29. 2x^3 + 128 30. 4t^2 - 25 31. 5a^3 - 45a^2 + 70a
32. 12k^2 - 36k + 27 33. a^3 - b^3 + 2a - 2b 34. 15x^2 + 11xy - 14y^2
35. 2m^2 - 10m - 48 36. 72r^3s^2 + 12r^2 - 24r^4s^2 37. 54y^2 - 24z^2
38. 100n^2r^2 + 30nr^3 - 50n^2r 39. 27p^{10} - 45p^9 - 252p^8 40. 16x^3z + 2y^3z

Solutions
1. 6(3x - 4) 2. 5x^2y(10x^3y + 7) 3. 9a^2b(4a^3 + 5a^2b^3 + 9b) 4. (x - y)(a + 5)
5. (t + 5)(t - 5) 6. 3(2x - 3y)(2x + 3y) 7. 3v(5A - 7t)(5A + 7t) 8. 2(x^2 + 16)(x + 4)(x - 4)
9. (p - 4)(p^2 + 4p + 16) 10. 3(a - 2b)(a^2 + 2ab + 4b^2) 11. 5y(x - 2y)(x^2 + 2xy + 4y^2)
12. (5x - 6y^2)(25x^2 + 30xy^2 + 36y^4) 13. 8(a + 2)(a^2 - 2a + 4) 14. 2(4x + 3)(16x^2 - 12x + 9)
15. u(10t + 3)(100t^2 - 30t + 9) 16. (4x + 7y^2)(16x^2 - 28xy^2 + 49y^4) 17. (a + 7)^2
18. (4u + 1)^2 19. (t - 6)^2 20. (4x - 3y)^2 21. (x - 6)(x + 2) 22. (3k - 2)(k + 2)
23. (4a + b)(2a - 3b) 24. (3m - 2n)(2m - 5n) 25. (3 + p)(2 + m) 26. (4 + s)(5 + 3t)
27. (2 - 3b)(2 - a) 28. (5 + x)(1 - y) 29. 2(x + 4)(x^2 - 4x + 16) 30. (2t + 5)(2t - 5)
31. 5a(a - 7)(a - 2) 32. 3(2k - 3)^2 33. (a - b)(a^2 + ab + b^2 + 2) 34. (5x + 7y)(3x - 2y)
35. 2(m - 8)(m + 3) 36. 12r^3(6rs^2 + 1 - 2r^2s^2) 37. 6(3y + 2z)(3y - 2z)
38. 10nr(10nr + 3n^2 - 5n) 39. 9p^8(3p + 7)(p - 4) 40. 2s(2x + y)(4x^2 - 2xy + y^2)